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Comparison of nonlinear and linearized oscillating mechanical systems

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1. Introduction

Vibrations are a natural phenomenon occurring during the operation of every machine. It is usually considered as a negative phenomenon resulting in additional loads and energy losses. Although, there are various machines and equipment which operation is based on oscillation [1, 4]. In this work, we analyzed several mechanisms, simplified as systems of point particles. We derived equations of motion for both nonlinear and linearized systems using methods of Lagrangian mechanics. This approach proved to be reliable and, in such cases, relatively simple.

2. Analyzed system

We analyzed several mechanisms consisting of connected pendulums and oscillators. We selected one, for which we provide deeper analysis. The system being solved consists of two coupled mathematical pendulums, with one being connected to horizontal oscillator.

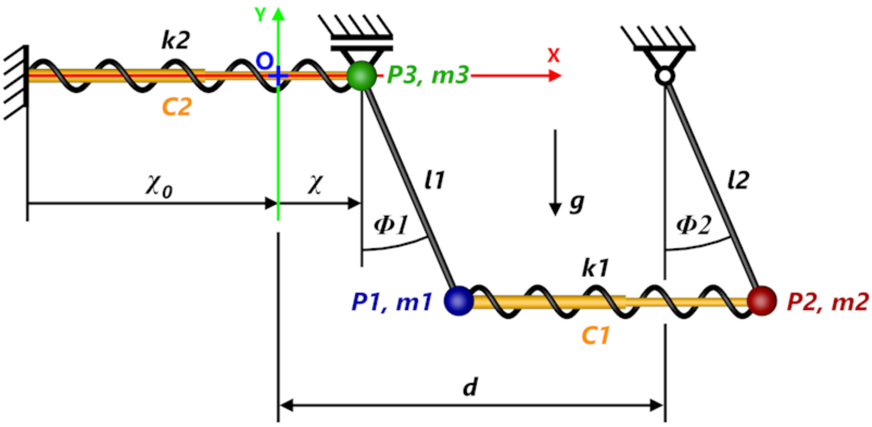

Fig. 1. Analyzed system

This system is described in Fig. 1, where m_1, m_2 and m_3 denotes masses of points P_1, P_2 and P_3 respectively, l_1, l_2 are lengths of massless strings and d is the distance between pendulum pivots, provided both lies on the X axis. Respective spring stiffnesses are k_1, k_2 and C_1, C_2 are coefficients of viscous damping. Standard gravity is denoted by g. Position of the system is fully described by tree generalized coordinates, i.e. two angles ϕ_1, ϕ_2 and deflection χ, with χ_0 being initial position of the oscillator. Coordinate system origin is in the point O.

3. Equations of motion

Lagrangian of the presented system, in the form $L = T - V$, where T is kinetic and V is potential energy of the system [2, 3], is

$$L = \left[\frac{m_1}{2}\left(\dot{\chi}^2 + l_1^2\dot{\phi}_1^2 + 2l_1\dot{\chi}\dot{\phi}_1\cos\phi_1\right) + \frac{m_2}{2}l_2^2\dot{\phi}_2^2 + \frac{m_3}{2}\dot{\chi}^2\right] - \frac{k_2}{2}\chi^2 - P + m_1 g l_1\cos\phi_1 + m_2 g l_2\cos\phi_2, \quad (1)$$

where the term in square brackets represents total kinetic energy of the system. The rest members then represent potential energy, with $P = k_1\left(\sqrt{Q} - s\right)^2/2$ being potential energy of spring between pendulums. The term Q is the distance between pendulum bops

$$Q = d^2 + l_1^2 + l_2^2 - 2d\chi + \chi^2 - 2l_1l_2\cos(\phi_1 - \phi_2) + 2l_1(\chi - d)\sin\phi_1 + 2l_2(d - \chi)\sin\phi_2\,. \quad (2)$$

Damping can be represented using Rayleigh functions $R_{D1} = C_1\dot{Q}^2/8Q$ for damper between pendulums and $R_{D2} = C_2\dot{\chi}^2/2$ for oscillator damper. The equations of motion have general form [2, 3]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = -\frac{\partial R_D}{\partial \dot{q}_k}, \quad (3)$$

where q_k represents generalized coordinates. Carrying out respective partial derivatives, we get system of 3 equations of motion:

$$\begin{aligned} F_{O\dot{\chi}} &= (m_1 + m_3)\ddot{\chi} + m_1 l_1\ddot{\phi}_1\cos\phi_1 - m_1 l_1\dot{\phi}_1^2\sin\phi_1 + k_2\chi + \frac{\partial P}{\partial \chi}, \\ F_{O\dot{\phi}_1} &= m_1 l_1\ddot{\chi}\cos\phi_1 + m_1 l_1^2\ddot{\phi}_1 + m_1 g l_1\sin\phi_1 + \frac{\partial P}{\partial \phi_1}, \\ F_{O\dot{\phi}_2} &= -m_2 l_2^2\ddot{\phi}_2 - m_2 g l_2\sin\phi_2 - \frac{\partial P}{\partial \phi_2}, \end{aligned} \quad (4)$$

with left-hand side being damping forces

$$F_{O\dot{\chi}} = -C_1\frac{\dot{Q}}{4Q}\frac{\partial \dot{Q}}{\partial \dot{\chi}} - C_2\dot{\chi}; \quad F_{O\dot{\phi}_1} = -C_1\frac{\dot{Q}}{4Q}\frac{\partial \dot{Q}}{\partial \dot{\phi}_1}; \quad F_{O\dot{\phi}_2} = -C_1\frac{\dot{Q}}{4Q}\frac{\partial \dot{Q}}{\partial \dot{\phi}_2}. \quad (5)$$

We then performed linearization using the first terms of Taylor series. Linearized Lagrangian (1) therefore gets the form

$$L = \frac{m_1}{2}\left(\dot{\chi}^2 + l_1^2\dot{\phi}_1^2 + 2l_1\dot{\chi}\dot{\phi}_1\right) + \frac{m_2}{2}l_2^2\dot{\phi}_2^2 + \frac{m_3}{2}\dot{\chi}^2 - \frac{k_2}{2}\chi^2 - \frac{k_1}{2}Q_L^2 - m_1 g l_1\phi_1^2 - m_2 g l_2\phi_1^2, \quad (6)$$

where $Q_L = l_2\phi_2 - (\chi + l_1\phi_1)$ is linearized deflection of spring between pendulums. For Rayleigh functions holds: $R_{D1} = C_1\dot{Q}_L^2/2$, $R_{D2} = C_2\dot{\chi}^2/2$, damping forces are then

$$\begin{aligned} F_{O\dot{\chi}} &= -(C_1 + C_2)\dot{\chi} - C_1 l_1\dot{\phi}_1 + C_2 l_2\dot{\phi}_2, \\ F_{O\dot{\phi}_1} &= -C_1 l_1\dot{\chi} - C_1 l_1^2\dot{\phi}_1 + C_1 l_1 l_2\dot{\phi}_2, \\ F_{O\dot{\phi}_2} &= C_1 l_2\dot{\chi} + C_1 l_1 l_2\dot{\phi}_1 - C_1 l_1^2\dot{\phi}_2. \end{aligned} \quad (7)$$

Finally, the system of linearized equations of motion is

$$\begin{aligned} F_{O\dot{\chi}} &= (m_1 + m_3)\ddot{\chi} + m_1 l_1\ddot{\phi}_1 + (k_1 + k_2)\chi + k_1(l_1\phi_1 - l_2\phi_2), \\ F_{O\dot{\phi}_1} &= m_1 l_1\ddot{\chi} + m_1 l_1^2\ddot{\phi}_1 + k_1 l_1\chi + \phi_1(k_1 l_1^2 + m_1 g l_1) - \phi_2 k_1 l_1 l_2, \\ F_{O\dot{\phi}_2} &= m_2 l_2^2\ddot{\phi}_2 - k_1 l_2\chi - k_1 l_1 l_2\phi_1 + \phi_2(k_1 l_2^2 + m_2 g l_2). \end{aligned} \quad (8)$$

System (8) was then solved in analytic fashion.

4. Results

Comparison of nonlinear and linear systems for small deflections, with initial conditions $\chi(0) = 0.1$ m, $\phi_1(0) = \phi_2(0) = 0$ rad, $\dot{\chi}(0) = 0$ m/s a $\phi_1(0) = \phi_2(0) = 0$ rad/s is on Fig. 2 (left). Results for large deflections, with initial conditions $\chi(0) = -0.75$ m, $\phi_1(0) = \phi_2(0) = 0$ rad, $\dot{\chi}(0) = 0$ m/s a $\dot{\phi}_1(0) = \dot{\phi}_2(0) = 0$ rad/s are on Fig. 2 (right). Parameters of the system were the same for both cases and are stated in Table 1. Although, we neglected coefficients of damping ($C_1 = C_2 = 0$) in case of large deflections.

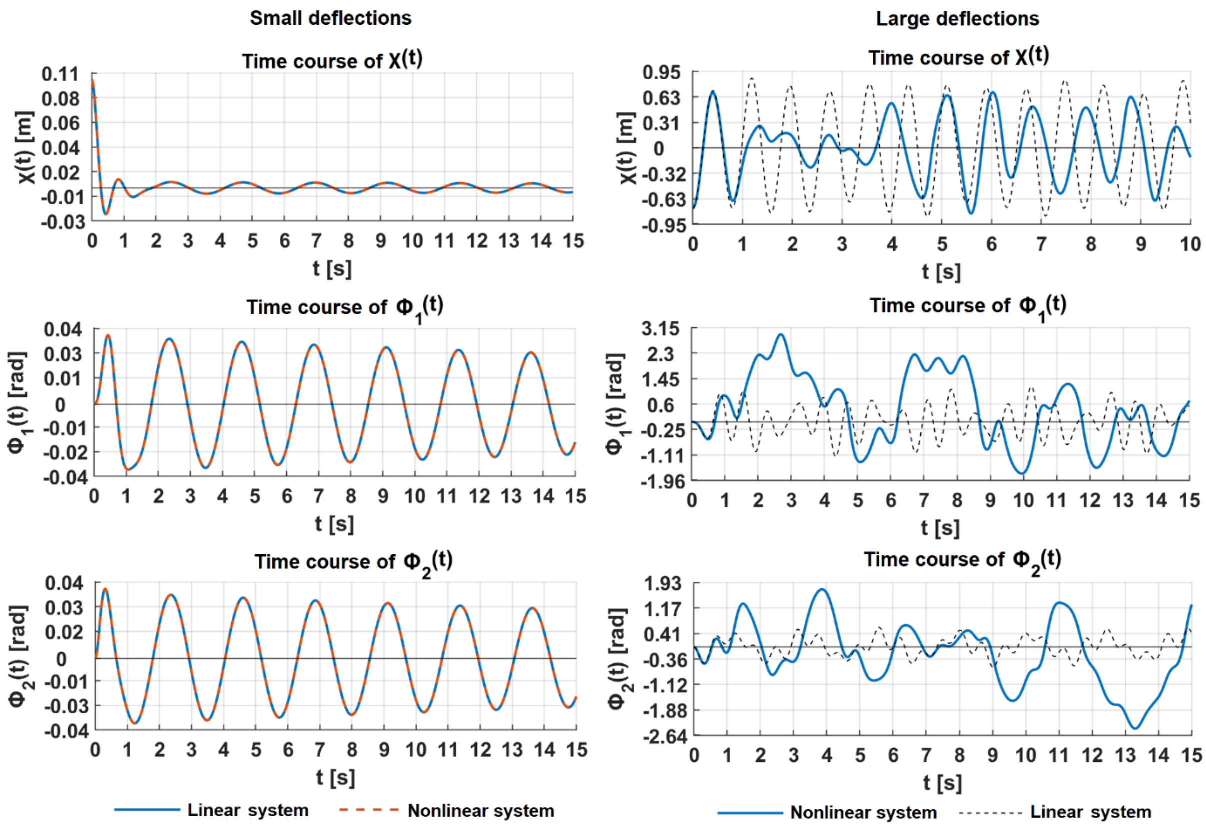

Fig. 2. Solution for small deflections (left) and large deflections (right)

As we can see, for small deflections, results are almost identical, however, in case of large deflections, nonlinear system becomes aperiodic and chaotic, with linear system being unable to represent this behavior.

Table 1. Parameters of the system

Masses	Lengths	Springs/Dampers		Standard gravity
		Linear case	Nonlinear case	
$m_1 = 1.50$ kg	$l_1 = 1.20$ m	$k_1 = 80.0$ N/m	(both cases)	$g = 9.81$ m/s^2
$m_2 = 2.00$ kg	$l_2 = 1.20$ m	$C_1 = 6.00$ kg/s	$C_1 = 0$	
$m_3 = 1.75$ kg	$d = 2.00$ m	$k_2 = 100$ N/m	(both cases)	
	$s = 2.00$ m	$C_2 = 10.0$ kg/s	$C_2 = 0$	

5. Conclusion

In case of small deflections, the solution of the linear system provided very good approximation of the analyzed mechanism and its behavior. This however does not hold for the case of large deflections, where nonlinear system becomes chaotic. Here, linear solution provides only linear combination of periodic functions and therefore cannot represent this chaotic behavior.

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