Dispersion characteristic of a 1D periodic structure

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Abstract **This paper presents a novel method for calculating the dispersion characteristic of 1D periodic generally lossy or radiating transmission lines. The method is applied to a leaky wave antenna based on the surface integrated waveguide. The calculation has been verified by measurement and by a numerical calculation.**

Keywords **1D periodic structure, transmission matrix, propagation constant, dispersion characteristic.**

I. INTRODUCTION

1D periodic structures are transmission lines loaded at periodic intervals with identical obstacles. There are two basic properties common to all periodic structures [1], namely passband/stopband characteristics and support for waves with phase velocity much less than the velocity of light. The passband/stopband properties are of interest for filter design. The low phase velocity is of basic importance in traveling-wave tubes. The slow wave allows the electron beam to interact with the wave.

The passband/stopband characteristics are described by the so-called dispersion characteristics, which represent the dependence of the frequency, or more precisely the product kd, as a function of the phase constant, or more precisely product γd; d is the line period, k is the free space propagation constant and γ is the generally complex propagation constant of the 1D periodically loaded line.

II. DISPERSION CHARACTERISTIC

The complex propagation constant of an infinitely long periodic 1D structure $\gamma = \alpha + j\beta$, where α is attenuation constant and β is phase constant, is calculated as [1],

$$\cos(\gamma d) = \frac{A+D}{2}\,, \qquad (1)$$

where A, D are elements of one cell ABCD transmission matrix. The periodic transmission line is composed of a chain of elementary cells that are tightly coupled. The transmission matrix $\mathbf{A}_c$ of the chain containing N cells is calculated or measured by any technique. The aim is to determine the ABCD matrix $\mathbf{A}$ of one "inner" cell, which includes the coupling with all neighbor cells

$$\mathbf{A} = \sqrt[N]{\mathbf{A}_c}\ . \qquad (2)$$

The N-th root of a matrix is calculated as

$$\mathbf{A} = \exp\left[\frac{1}{N}\ln(\mathbf{A}_c)\right], \qquad (3)$$

the exponential function is defined by the power sum

$$\exp(\mathbf{A}) = \sum_{k=0}^{\infty}\frac{1}{k!}\mathbf{A}^k\ . \qquad (4)$$

The logarithm of a matrix is calculated using its diagonalized matrix $\mathbf{A}$'

$$\ln\mathbf{A}_c = \mathbf{V}(\ln\mathbf{\Lambda})\mathbf{V}^{-1},\quad \mathbf{\Lambda} = \mathbf{V}^{-1}\mathbf{A}_c\mathbf{V}\,, \qquad (5)$$

where $\mathbf{V}$ is a matrix with its columns composed of eigen vectors of $\mathbf{A}_c$ and $\mathbf{\Lambda}$ is a diagonal matrix of eigen values of $\mathbf{A}_c$. This procedure was applied to the surface integrated waveguide antenna based on the substrate integrated waveguide composite right/left-handed transmission line presented in [2]. The dispersion characteristics are plotted in Fig. 1 for increasing number of unit cells N. The convergence is relatively fast, and it is sufficient to use only about 9 cells. To assure unambiguity of the solution, we have to guess independently the frequencies of the zero values of the propagation constant, or we have to combine the solution for two subsequent values of N.

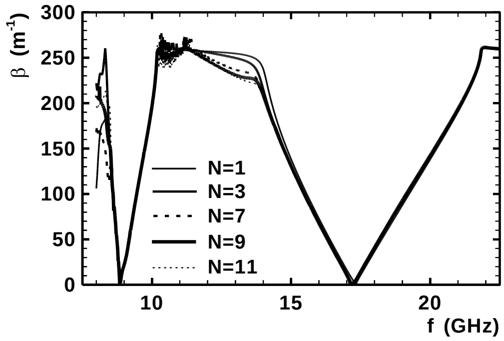

Fig. 1 Dispersion characteristic - real part of the phase constant - of the antenna [2] calculated for different values of N.

III. CONCLUSION

A method for calculating the dispersion characteristics of a 1D periodic transmission line has been derived. The method is applicable to generally lossy and radiating structures. The example is the characteristic of a leaky wave antenna. The method was verified by numerical simulation and by measurement. More examples will be presented at the conference.

IV. ACKNOWLEDGEMENTS

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V. REFERENCES

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