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Geometrically nonlinear thermoelastic numerical analysis of an actuator made of nylon springs

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This article presents the results of a geometrically nonlinear elastic and thermoelastic analysis of an actuator assembled from two nylon springs in the shape of a Von Mises rod structure. The springs are prestressed by compressive axial force and are characterized by negative thermal expansion. Based on the Updated Lagrange Formulation (ULF) of finite displacements, a calculation model of the finite element method is compiled using a special finite element of a nylon spring [2]. The mechanical and thermomechanical parameters of the springs, which are obtained by measuring them, enter the mathematical model. The stiffness matrix of the actuator is compiled using the classic approach of the finite element method and consists of linear and geometric stiffness. The separated equation for calculating the displacement of the common node of the actuator springs is solved by the Newton incremental method. The results of the numerical method are compared with the results of the measurement of the real actuator, whereas a very good agreement was reached. The actuator in the form of a symmetrical Von Mises rod system in the initial unloaded state is shown in Fig. 1a).

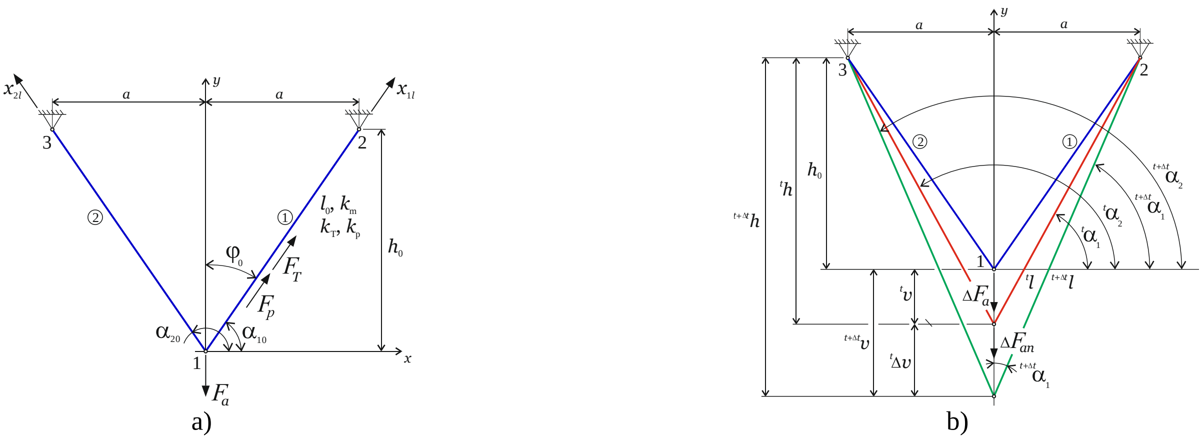

Fig. 1. Actuator geometry: a) in initial position, b) in deformed position for time step Δt

The actuator represents the connection of two identical pre-stressed nylon springs with negative thermal expansion in the global plane coordinate system x, y. Springs are defined by initial length l_0, initial linear stiffnesses k_m and k_p, compressive preload F_p, negative thermal expansion k_T and initial angle φ_0. Local axes of springs no. 1 and no. 2 are labeled as x_{1l} and x_{2l}. These local axes form an initial angle of α_{10} , respectively. α_{20}, with the global axis x. The pressure prestress, which is created by the technology of winding the spring by twisting the nylon fiber, compresses the coils of the springs in an unloaded state. Zero displacements (fixed joints) are prescribed in nodes 2 and 3. The tensile vertical mechanical force F_a (caused, for example, by the weight of a mass) acts in node 1 in the opposite direction of the coordinate axis

y. This force, which must be greater than the resulting compressive force of the spring prestress in the actuator, will cause the final vertical displacements of point 1 through a large change in the initial length of the springs - see Fig 1b). Fig. 2a) represents the result of measuring the tensile characteristics of the springs as well as the actuator. The measured tensile characteristics of both springs are almost identical and consist of three parts. In its first part (I), the negative prestress F_p with stiffness k_p is overcome. The extension of shorter springs tends to be small in this area and is caused by the rigid movement of the coils, but it can also be more pronounced for springs with a larger number of coils. This part of the tensile characteristic can be minimized by initially training the springs (several stretches). In the second part, a small linear region with stiffness k_m follows. In the third part, the stiffness of the springs begins to increase non-linearly to the point where the coils of the spring begin to gradually unwind. The nonlinear stiffness of the spring in the reference configuration t is ${}^{t}k_{NL} = k_m + {}^{t}k_G$, while the geometric stiffness ${}^{t}k_G = {}^{t}N / {}^{t}l$ with the axial force in the spring ${}^{t}N$ and reference spring length ${}^{t}l$ [2]. The tensile characteristic of the actuator has a similar course. Simplified (linearized) characteristic of tensile test of springs, or of the actuator is shown in Figs. 2b), or 2c).

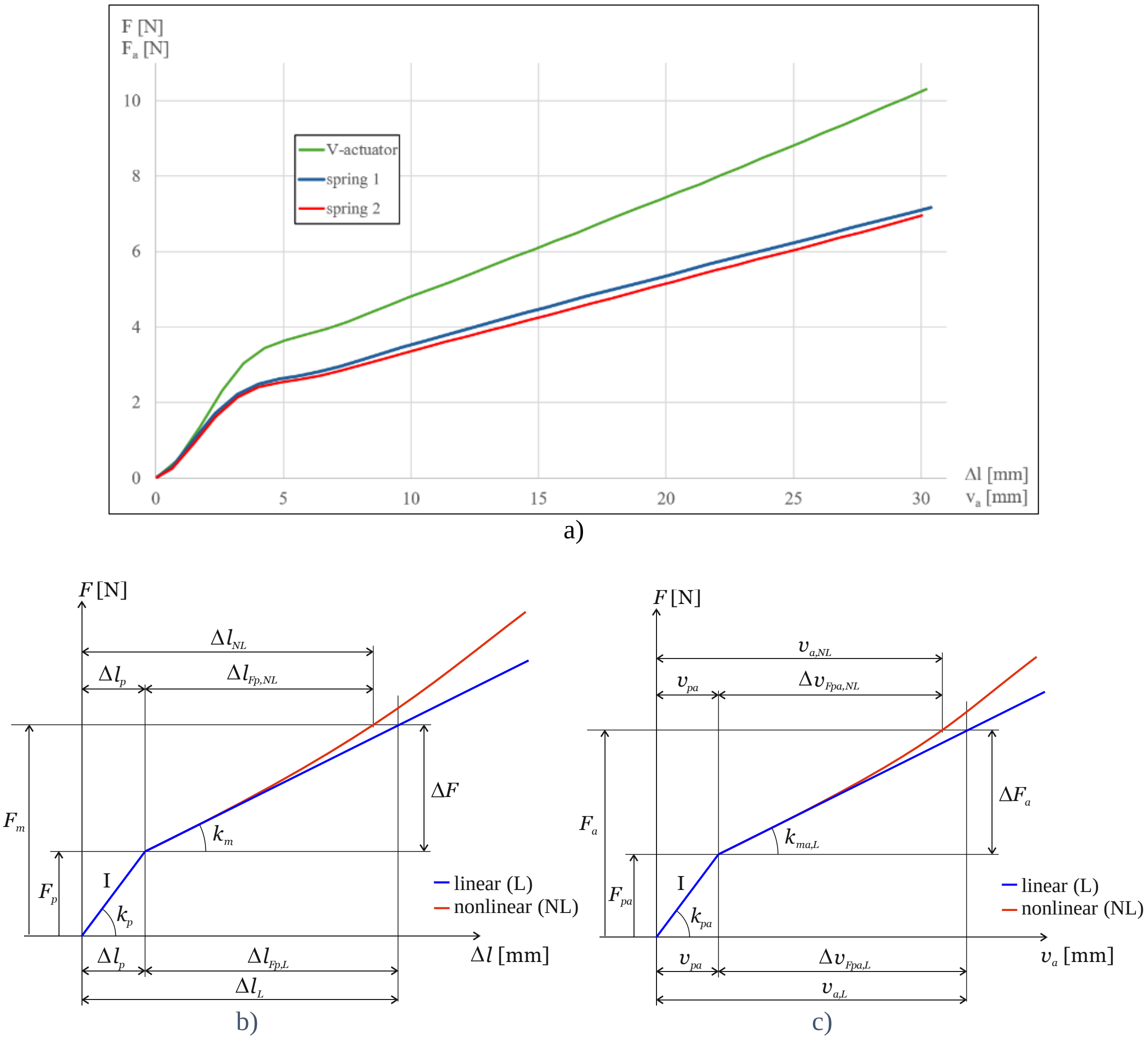

Fig. 2. Tensile characteristics of individual springs and actuator: a) measured, b) simplified for spring, c) simplified for actuator

By warming both mechanically stretched springs from the ambient temperature by a temperature difference ΔT, action can be triggered in the form of lifting the weight up, maximally to the initial position. For small changes in the length of the springs, the linear theory of continuum mechanics can be applied. However, from the point of view of the need for a more significant action intervention of the actuator, larger changes in the length of the springs are required. For this reason, as shown in [2], it is necessary to apply the geometrically nonlinear theory of continuum mechanics for computational simulation of work characteristic. Based on the Updated Lagrange Formulation (ULF [1]) of the deformation movement of the continuum, its reference configuration is located at time *t*, (Fig. 1b). The equations of motion of the continuum can be solved by the incremental Newton method or the incremental-iterative Newton-Raphson method. In our case, Newton's incremental method is applied. The increase in the mechanical and thermal load of the actuator, $\Delta F_{an} = \left(F_a - 2\left(F_p + F_T\right)\right)\frac{h_0}{l_0}/nmax$, in the reference configuration causes an increase in the vertical increment of the displacement ${}^{t}\Delta v$ at point 1 to the configuration at time *t*+Δ*t*. The size of the load increment depends on the selected number of load steps *nmax*, while we will assume its same size in all load steps. $F_T = k_m k_T \Delta T$ is the thermomechanical force in the spring caused by heating Δ*T*. Here, time is only a formal parameter representing the position of the body in the incremental solution method. According to Fig. 1b), the spring reference parameters are spring length ${}^{t}l$, spring tensile stiffness ${}^{t}k_{NL}$, spring inclination angles ${}^{t}\alpha_1$ and ${}^{t}\alpha_2$. The reference configuration of the actuator gradually changes during the incremental solution method up to the position caused by the total magnitude of the mechanical and thermal load.

The procedure for compiling the incremental equations for calculating the increments of the primary quantities of the geometrically nonlinear elastostatics of the Von Mises actuator, assembled from two nylon springs with compressive preload and negative thermal expansion, is based on the principle of the finite element method. Each spring is an innovative two-node NTCS (Nylon Twisted Coiled Spring) LINK-type finite element with two degrees of freedom at each nodal point in the plane global coordinate system (x,y) [2]. The linearized local NTCS stiffness matrix of finite element No. 1 and 2 in the planar global coordinate system has a dimension of 4×4. After its expansion, this matrix has the dimension NDOF×NDOF, where NDOF = 6 is the total number of degrees of freedom of the nodal points of the actuator. The process of constructing the global actuator stiffness matrix ${}^{t}K_{NL}$ is standard within the finite element method procedure and is therefore not detailed in this paper. Considering the given boundary conditions and actuator symmetry, Fig. 1b), the only non-zero increase in displacement is the vertical elastic displacement of node 1 in direction of the y axis, ${}^{t}\Delta v$. Its increment in the given loading step $i \in (2, nmax)$ for nonlinear part of the tensile characteristic is: ${}^{t}\Delta v = \Delta F_{an}/{}^{t}k_{NL}$. The relevant nonlinear stiffness of the actuator ${}^{t}k_{NL}$ is the sum of the linear stiffness $k_{ma,L} = 2k_m\left(\frac{h_0}{l_0}\right)^2$ and of the geometric stiffness of the actuator ${}^{t}k_{Ga} = \frac{{}^{t}N}{{}^{t}l}\left(\frac{{}^{t}h}{{}^{t}l}\right)^2$ [2]. Since the total vertical displacement of node 1 at time *t*, ${}^{t}v$, is known, then the total vertical displacement in the configuration $t + \Delta t$ is ${}^{t+\Delta t}v = {}^{t}v + {}^{t}\Delta v$. This cycle is repeated for $nmax$-load steps. However, in the first load step, $i = 1$, ${}^{t}v = {}^{t}\Delta v = \Delta F_{an}/2k_m\left(\frac{h_0}{l_0}\right)^2$. The procedure for compiling the finite element nonlinear equations of the actuator, as well as the algorithm for their solution, was programmed by the authors of this paper in the MATHEMATICA software environment [3] and will be presented in detail in the conference presentation. The accuracy of the solution depends on the number of load steps *nmax*, but convergent results can be achieved already at $nmax = 30$. According to Fig. 2c) for the first part of the tensile characteristic, $v_{pa} = F_{pa}/k_{pa}$ is the rigid displacement of point 1 of the actuator in the first part of the measured characteristic of the actuator. $k_{pa} = 2k_p\left(\frac{h_0}{l_0}\right)^2$ is the stiffness of the actuator in this part of the characteristic and $F_{pa} = 2F_p\frac{h_0}{l_0}$ is the preload force of the actuator. If we denote the resulting vertical nonlinear thermoelastic displacement of point 1 in step $i = nmax$ as ${}^{t+\Delta t}v \equiv v_{Fpa,NL}$, then the resulting vertical nonlinear displacement of point 1 of the actuator is $v_{a,NL} = v_{pa} + v_{Fpa,NL}$. For $nmax = 1$, a linear solution $v_{a,L}$ can be obtained.

In the numerical and physical model, the actuator in the form of a Von Mises rod system is assembled in the global coordinate system x, y from two identical springs with an initial length $l_0 = 85$mm and an angle $\varphi_0 = 37°$. From the results of measuring the characteristics of the springs shown in Fig. 2a) and for the simplified characteristics of Figs. 2b) and 2c), the required spring parameters are determined: prestress force $F_p = 2.45$ N, stiffness $k_m = 0.180$ N/mm, stiffness $k_p = 0.61$ N/mm, prestress force $F_{pa} = 3.914$ N, vertical displacement $v_{pa} = 5.03$ mm. Table 1 shows the results of the elastostatic solution of the actuator loaded with the mechanical force $F_a \in< 3.94, 30.00 >$ N. The geometrically nonlinear analysis was performed with $nmax = 30$, and for linear analysis $i = 1$ was set.

Table 1. Results of elastostatic linear and non-linear analysis and measurements of the actuator

F_a	ΔF_a	v_{pa}	$v_{Fpa,L}$	$v_{a,L}$	$v_{Fpa,NL}$	$v_{a,NL}$	$v_{a,exp}$	Δl_L	Δl_{NL}
3.94	0.026	5.03	0.11	5.14	0.11	5.14	6.68	0.16	0.089
5.0	1.086	5.03	4.73	9.76	4.63	9.66	10.75	3.82	3.74
6.06	2.146	5.03	9.34	14.37	8.97	14.00	14.04	7.63	7.32
6.94	3.026	5.03	13.17	18.20	12.45	17.48	18.09	10.85	10.24
8.01	4.096	5.03	17.83	22.86	16.55	21.58	22.15	14.82	13.73
9.15	5.236	5.03	22.79	27.82	20.77	25.80	26.19	19.11	17.36
10.07	6.156	5.03	26.79	31.82	24.07	29.10	29.41	22.62	20.23
15.00	11.086	5.03	48.26	53.29	40.45	45.48	-	41.91	34.81
20.00	16.086	5.03	70.02	75.05	55.30	60.33	-	62.09	48.39
25.00	21.086	5.03	91.79	96.82	68.83	73.86	-	82.671	60.97
30.00	26.086	5.03	113.55	118.58	81.33	86.36	-	103.52	72.75

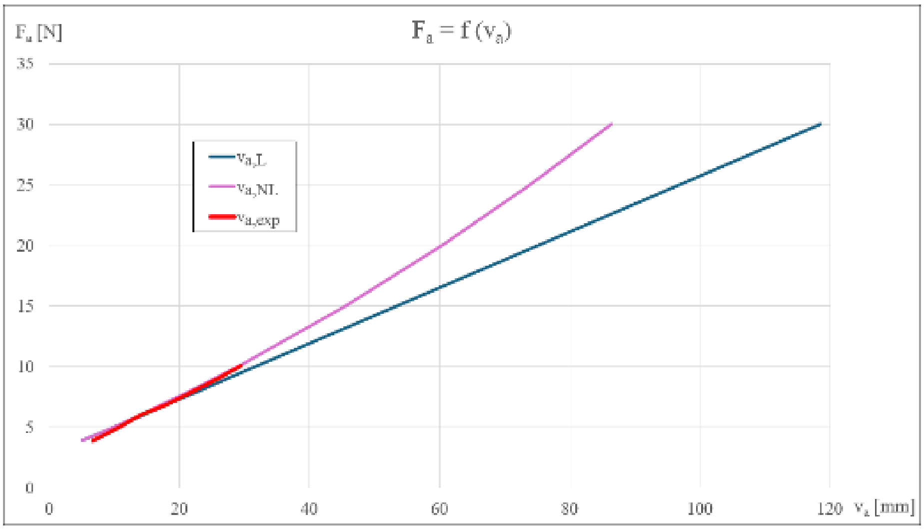

Fig. 3. Graphical comparison of the calculated and measured results for the actuator

As is evident from Table 1 and Fig. 3, the agreement of the nonlinear calculation and measurement results is very good. The capabilities of the measuring device did not allow measuring larger deformations of the actuator. However, even from the measured measurement range, the trend of the measurement results to match the results of the nonlinear analysis is obvious.

From the measurement, when the individual spring is loaded with a force $F_a = 5.55$ N (the individual springs are loaded by force 3.47 N), the thermal expansion coefficient of the spring is $k_T = 0.23$ mm/°C, and with force $F_a = 20.0$ (the individual springs are loaded by force 12.5 N) is $k_T = 0.06$ mm/°C. The measurement is performed in a thermal chamber.

The results of linear and non-linear thermoelastic analysis for several temperature differences ΔT are presented in Tables 2 and 3. The thermal stroke sT, of the actuator are marked by italic-style font. Difference between linear and nonlinear results is evident.

Table 2. Thermoelastic displacements for $F_a = 5.55$ N

ΔT[°C]	0	5	10	15	20	22	24
$v_{Fpa,L}/sT$	7.12	5.64/*1.48*	4.2/*2.92*	2.76/*4.36*	1.32/*5.80*	0.74/*6.38*	0.17/*6.95*
$v_{Fpa,NL}/sT$	6.90	5.50/*1.4*	4.12/*2.78*	2.72/*4.18*	1.31/*5.59*	0.74/*6.16*	0.17/*6.73*

Table 3. Thermoelastic displacements for $F_a = 20$ N

ΔT[°C]	0	10	15	20	50	100
$v_{Fpa,L}/sT$	70.22	69.18/*1.04*	68.76/*1.46*	68.34/*1.86*	65.83/*4.39*	61.63/*8.49*
$v_{Fpa,NL}/sT$	55.30	54.75/*0.55*	54.48/*0.82*	54.21/*1.09*	52.55/*2.75*	49.76/*5.54*

As can be seen in Table 2, for relatively small elastic extensions of the springs, the difference between the linear and non-linear solutions is small. Due to the lower tensile strength, the coefficient of thermal expansion is greater, which results in a greater temperature reduction even (thermal stroke) with a small warming. As shown in Table 3, with larger tensile forces, the coefficient of thermal stroke is smaller, which causes larger thermal shortening only with relatively high heating of the springs. However, the effective lifting force is significantly greater. The designed and analysed actuator can be used in practice to develop a significant action intervention in the form of action force or action movement. The original computational model can be used in practice in the design of real actuators or artificial muscles assembled from NTCS springs with negative thermal expansion.

Acknowledgements

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