

# Modal analysis of beam with piezoelectric sensors and actuators

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## Abstract

One dimensional finite element is developed for the analysis of structures with applied piezoelectric sensors and actuators, i.e. smart structures, mechanical behavior of which can be controlled in real-time. The element is based on Euler-Bernoulli theory and it assumes bilinear distribution of electric field potential. Mathematical model was implemented in MATLAB environment. Sensitivity analysis is carried out for the case of modal analysis with and without piezo patches.

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## 1. Introduction

Light-weight structures are nowadays necessary components in modern state-of-the-art products in all sorts of industries. The increasing requirements on structural performance call for the usage of embedded sensors and actuators, resulting in the construction of so-called smart or adaptive structures that can thus respond to loading conditions in real time. This enables for instance to suppress vibrations or to adapt the desired shape, provided that proper electronic control circuits are applied.

Since the first finite element implementation of the piezoelectric phenomenon by Allik and Hughes [1] in 1970, many researches have 'equipped' the standard structural finite elements with the piezoelectric capability to simulate the piezoelectric effect – sometimes using the similarity to the theory of thermo-elasticity. These early models concerned mainly 3D-solid elements, which are not suited for efficient analysis of laminated shell structures. In the recent years, the piezoelectric, beam, plate and shell elements are used more frequently. Cen et al [3], for example, developed a four-node plate element for laminated structures based on first-order shear deformation theory while Lee et al. [7] introduced a nine-node assumed strain element allowing, unlike other elements, for variable thickness. Hybrid laminated piezo plates are studied by Mitchell and Reddy [8] using higher-order shear deformation theory and layerwise approach for electric potential. Saravanos et al. [9] study dynamic behavior of smart laminated plates using the layerwise approach. Tzou et al. [11] investigate the control of smart conical shells using triangular finite elements. Kögl and Bucalem [6] introduced a MITC based element suitable for modeling of moderately thick sandwich smart structures. They stress the importance of quadratic variation of electric potential across the layer thickness to accurately model the electric field. Other various finite element approaches are summarized for example in the survey by Benjeddou [2]. Zhou et al. [12] study free vibrations of piezoelectric bimorphs by means of

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