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DYNAMICS OF ROTATING SYSTEMS WITH ROLLING ELEMENT BEARINGS

M. Hajžman[1]

Abstract: This paper deals with a suitable approach to the computational modelling of rolling element bearings in the framework of rotating systems dynamics. Various alternative approaches are briefly summarized and a chosen moderately complex approach is described in more detail. The presented bearing model respects real number of rolling bodies and roller contact forces acting between the journals and the outer housing.

Keywords: equations of motion; rolling element bearing; force modelling; contact

1 Introduction

All rotating bodies as parts of various mechanical systems should be sufficiently supported and fixed to a frame, a housing or to a stator. Axial and radial types of bearings can be mainly distinguished in order to support rotors in two main directions. Further division is based on the used physical principle — it means rolling element bearings and sliding bearings (oil-film bearings, magnetic bearings, aero-static bearings etc.).

There exist different approaches to the modelling of rolling element bearings that are suitable for different applications. The simplest one is characterized by in-line stiffness in static force direction and the lateral stiffness [1] and is modelled using two springs in these main perpendicular directions. Then several moderate complex models based in the force description of particular rolling elements can be found, while one of this family is described in this paper. Comprehensive models are based on the multibody dynamics approaches where models contains rolling elements considered as rigid or flexible bodies with real contact conditions and with their own degrees of freedom. Specialized rolling element bearings are created using the finite element method and can be utilized for detailed contact and fatigue analyses.

2 Rolling element bearing model

The force vector representing the rolling element contact forces [1] can be expressed as

$$F_{i,j} = \left(\frac{\Delta_{i,j}}{C_i}\right)^{n_i} H(\Delta_{i,j}), \quad F_{i,j}^{ax} = \left(\frac{\Delta_{i,j}^{ax}}{C_i^{ax}}\right)^{n_i^{ax}} H(\Delta_{i,j}^{ax}) \tag{1}$$

and is given by radial $\Delta_{i,j}$ or axial $\Delta_{i,j}^{ax}$ deflections for the rolling bodies (Fig. 1). These deflections arise from deformation at the contact points of the rolling elements and the races. Parameters n_i, C_i for radial and n_i^{ax}, C_i^{ax} for axial bearings depend on geometrical parameters, elastic modulus and Poisson's ratio of the bearing components. Heaviside functions in (1) correct the contact forces when deflections are negative. To obtain the deflections we assume that the rigid inner race with centre $\mathrm{S_i}$ is displaced by vector $\boldsymbol{q}_i = [u_i\ v_i\ w_i\ \varphi_i\ \vartheta_i\ \psi_i]^T$ and flexible outer race fixed with housing in contact points $\mathrm{H_{i,j}}$ is displaced by amounts $\boldsymbol{q}_j^S = [u_j\ v_j\ w_j]^T$ in directions of coordinate axis x, y, z (see Fig. 1). Then radial and axial rolling elements deflections are

$$\Delta_{i,j} = \boldsymbol{t}_{i,j}^T \boldsymbol{q}_i - \boldsymbol{e}_{i,j}^T \boldsymbol{q}_j^S - \gamma_i, \quad \Delta_{i,j}^{ax} = \boldsymbol{t}_{i,j\,ax}^T \boldsymbol{q}_i - \boldsymbol{e}_{i,j\,ax}^T \boldsymbol{q}_j^S - \gamma_i^{ax}, \tag{2}$$

where positive γ_i (γ_i^{ax}) expresses radial (axial) play between the races and negative γ_i (γ_i^{ax}) correspond to preloaded bearings.

[1] Michal Hajžman; Department of Applied Mechanics, Výzkumný a zkušební ústav Plzeň s.r.o.; hajzman@vzuplzen.cz

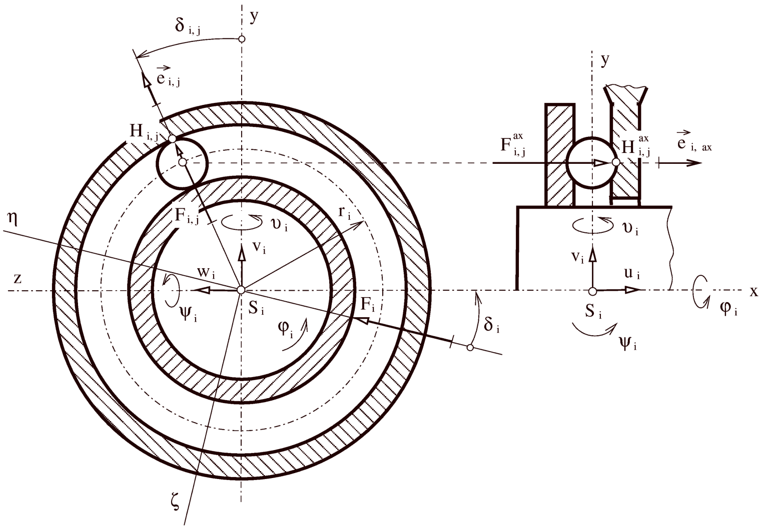

Figure 1: Scheme of a roller bearing with a particular rolling element.

The vectors in equations (2) are

$$\boldsymbol{t}_{i,j}^T = [0 \;\cos\delta_{i,j}\;\sin\delta_{i,j}\;0\;0\;0], \quad \boldsymbol{e}_{i,j}^T = [0\;\cos\delta_{i,j}\;\sin\delta_{i,j}], \tag{3}$$

$$\boldsymbol{t}_{i,j\,ax}^T = [1\;0\;0\;0\;r_i\sin\delta_{i,j}\; -r_i\cos\delta_{i,j}], \quad \boldsymbol{e}_{i,j\,ax}^T = [1\;0\;0]. \tag{4}$$

Vector $\boldsymbol{f}^B$ in equation of motion for a studied rotating system can be expressed in form

$$\boldsymbol{f}^B = -\sum_i \sum_j \left(\tilde{\boldsymbol{t}}_{i,j} F_{i,j} + \tilde{\boldsymbol{t}}_{i,j\,ax} F_{i,j}^{ax}\right). \tag{5}$$

The position of vectors $\boldsymbol{t}_{i,j}$, $\boldsymbol{t}_{i,j\,ax}$ in extended vectors of dimension equal to degrees of freedom n

$$\tilde{\boldsymbol{t}}_{i,j} = [\ldots\;\boldsymbol{t}_{i,j}^T\;\ldots]^T, \quad \tilde{\boldsymbol{t}}_{i,j\,ax} = [\ldots\;\boldsymbol{t}_{i,j\,ax}^T\;\ldots]^T,$$

corresponds to the position of the bearing centre displacements in the general coordinate vector $\boldsymbol{q}$.

3 Conclusion

The approach to the detailed force modelling of rolling element bearings was introduced in this paper. The overall motion of the rotating system supported by rolling element bearings is then given by

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + (\boldsymbol{B} + \omega\boldsymbol{G})\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \boldsymbol{f}^E(t) + \boldsymbol{f}^B, \tag{6}$$

where mass, damping and stiffness matrices $\boldsymbol{M}$, $\boldsymbol{B}$, $\boldsymbol{K}$ are symmetrical of order n and gyroscopic matrix $\boldsymbol{G}$ is skew symmetrical. Vector $\boldsymbol{f}^E(t)$ describes force excitation. Linearization of bearing forces can be performed for many real cases.

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